# SuperOPF Research Roadmap

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# 1 Background

## 1.1 The Need

There are a wide range of activities in the power systems area that depend critically on the availability of tools which enable decision-makers to properly allocate and value system resources, including shared public goods such as reliability. The following is a very incomplete list of some of these activities.

- electricity markets
  - design and operation of markets
  - markets for energy, capacity, ancillary services
  - all time scales from real-time to multi-year forward markets
- power grid operations
  - unit commitment
  - unit dispatch
  - maintenance scheduling
- regulatory oversight
  - market monitoring
  - setting and monitoring reliability standards
  - evaluating impacts of environmental regulations
- resource planning
  - optimal investment
  - reliability studies
  - evaluation of economic and reliability impacts of changes in technology: wind, solar, PHEV, DER, CHP, smart grid

Current state-of-the-art tools typically break the relevant optimization problems down into sequences of sub-problems, often using DC approximations to model the transmission network and replacing voltage and adequacy requirements with corresponding proxy constraints. This approach may be adequate to find a solution in which the allocations approximate the optimal, but the prices are often distorted, especially when the system is stressed. It is precisely under stressed conditions when correct prices are most informative for identifying the location of existing weaknesses in the network, what equipment needs to be added or upgraded and the net benefits of these upgrades. Using proxy limits for planning system adequacy, such as reserve margins to ensure adequate generation capacity, tends to obscure the real weaknesses in the system.

### 1.2 The Objective

The objective of the SuperOPF project is to develop a framework that will provide proper allocation and valuation of resources through true co-optimization across multiple scenarios. Instead of solving a sequence of simpler and approximate sub-problems, the SuperOPF approach combines as much as possible into a single mathematical programming framework, with a full AC network and simultaneous co-optimization across multiple scenarios with stochastic costs.

This effort involves development of the problem formulations, implementation of research grade software codes, and testing of the methods and algorithms on a range of case studies to demonstrate their added value over currently available tools.

## 2 Roadmap

The strategy for developing the SuperOPF can be structured into three levels as illustrated in Figure 1.

### 2.1 Level 1 – Extensible Optimal Power Flow

The foundation of the SuperOPF is an extensible optimal power flow formulation, consisting of a standard AC OPF with certain user supplied extensions. The traditional formulation minimizes the cost of generation subject to the nodal real and reactive power balance equations and the usual limits on voltage magnitudes, branch flow limits and generator outputs. It can be expressed in the following form.

$$\min_x f(x) \tag{1}$$

subject to

$$g(x) = 0 \tag{2}$$

$$h(x) \leq 0 \tag{3}$$

$$x_{\min} \leq x \leq x_{\max} \tag{4}$$

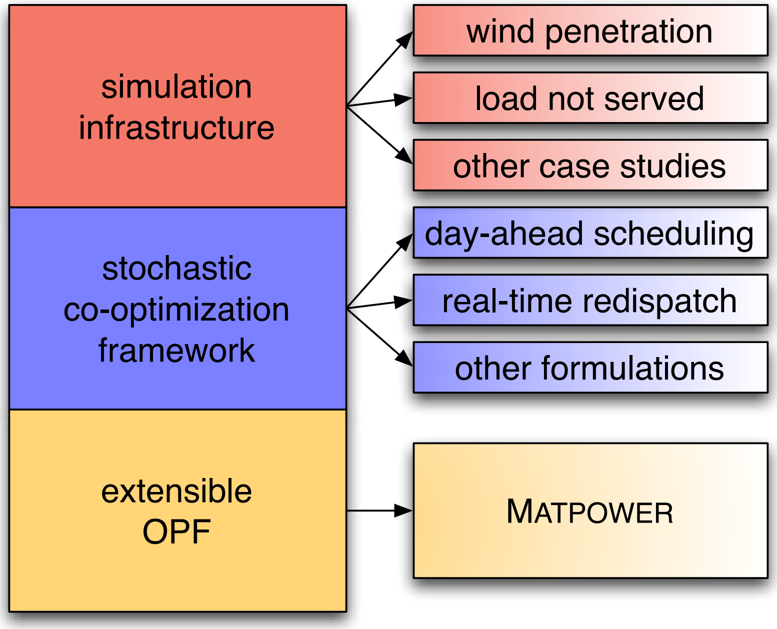


Figure 1: Three Level Structure

This is a general non-linear constrained optimization problem, with both non-linear costs and constraints. The optimization variable $x$ is defined in terms of the $n_b \times 1$ vectors of bus voltage angles $\Theta$ and magnitudes $V$ and the $n_g \times 1$ vectors of generator (generalized to include dispatchable loads) real and reactive power injections $P$ and $Q$ as follows.

$$x = \begin{bmatrix} \Theta \\ V \\ P \\ Q \end{bmatrix} \tag{5}$$

The objective function (1) is simply a summation of individual polynomial cost functions $f_P^i$ and $f_Q^i$ of real and reactive power injections, respectively, for each generator.

$$\min_{\Theta,V,P,Q} \sum_{i=1}^{n_g} \left[ f_P^i(p_i) + f_Q^i(q_i) \right] \tag{6}$$

The equality constraints (2) consist of two sets of $n_b$ non-linear nodal power balance equations, one for real power and one for reactive power.

$$g_P(\Theta, V, P) = 0 \tag{7}$$

$$g_Q(\Theta, V, Q) = 0 \tag{8}$$

The inequality constraints (3) consist of two sets of $n_l$ branch flow limits as nonlinear functions of the bus voltage angles and magnitudes, one for the *from* end and one for the *to* end of each branch.

$$h_f(\Theta, V) \leq 0 \tag{9}$$
$$h_t(\Theta, V) \leq 0 \tag{10}$$

The variable limits (4) include an equality limited reference bus angle and upper and lower limits on all bus voltage magnitudes and real and reactive generator injections.

$$\theta_{\text{ref}} \leq \theta_i \leq \theta_{\text{ref}}, \qquad i = i_{\text{ref}} \tag{11}$$
$$v_i^{\min} \leq v_i \leq v_i^{\max}, \qquad i = 1 \dots n_b \tag{12}$$
$$p_i^{\min} \leq p_i \leq p_i^{\max}, \qquad i = 1 \dots n_g \tag{13}$$
$$q_i^{\min} \leq q_i \leq q_i^{\max}, \qquad i = 1 \dots n_g \tag{14}$$

Here $i_{\text{ref}}$ denotes the index of the reference bus and $\theta_{\text{ref}}$ is the reference angle.

The extensions to the standard formulation that form the basis for the SuperOPF framework include additional optional user-defined costs $f_u$, linear constraints, and variables $z$. These augment the problem formulation as follows.

$$\min_{x,z} f(x) + f_u(x, z) \tag{15}$$

subject to

$$g(x) = 0 \tag{16}$$
$$h(x) \leq 0 \tag{17}$$
$$x_{\min} \leq x \leq x_{\max} \tag{18}$$
$$l \leq A \begin{bmatrix} x \\ z \end{bmatrix} \leq u \tag{19}$$
$$z_{\min} \leq z \leq z_{\max} \tag{20}$$

The additional user-supplied cost term in (15)

$$f_u(x, z) = \frac{1}{2} w^T H w + C w \tag{21}$$

is a general quadratic cost on a vector $w$ that is derived from the optimization variables in two steps. First, a linear combination $r$ of the optimization variables is defined by

$$r = N \begin{bmatrix} x \\ z \end{bmatrix}, \tag{22}$$

then a translation, a dead zone, and individual scalar functions, chosen by the user out of a predefined library set, are applied to each of the elements in $r$ to yield $w$. This way, many classes of functions can be applied to all of the optimization variables in the problem.

These extensions are used internally by MATPOWER to add a number of additional features to the standard OPF, including

- piecewise-linear costs on generation
- generator P-Q capability limits
- voltage angle difference limits
- price sensitive (dispatchable/interruptible) demands

They are also available to be used by higher level programs, such as those used to implement the stochastic co-optimization framework of the SuperOPF.

#### 2.1.1 Current Status

MATPOWER is an open-source power system simluation package that is freely downloadable from [1]. The capabilities described above are available for the AC OPF in version 3.2, which is currently available. They have been enhanced and extended to the DC OPF as well in a development version which will be made publicly available soon.

There are a number of different solvers available for the AC OPF problem in MATPOWER, including several that are distributed separately in the optional MINOPF [2] and TSPOPF [3] packages. These are implemented as Matlab extensions, called MEX files, written in C or Fortran code. There are three primarily that have been used for SuperOPF applications:

- PDIPM - primal dual interior point method
- SCPDIPM - step-controlled PDIPM
- MINOS

Of these solvers, the primal-dual interior point solvers in TSPOPF described in [4] have proven to be the most robust and the highest performance on many of the problems encountered in the SuperOPF cases studies to date.

However, the current implementation does not explicitly handle user-supplied linear equality constraints, resulting in robustness issues under certain circumstances.

The MINOS solver also had difficulty solving some of the co-optimization problems. In addition, MEX file implementations have their own set of issues with respect to code robustness related to minor version incompatibilities between the 3 environments used to build Matlab, to build the MEX file, and to run the simulation. The simulation software robustness in the end can be sensitive to the versions of MATLAB®, compiler, libraries, operating system, etc. making MEX files rather burdensome to maintain in a typical computing environment where each component is kept up-to-date for security and other reasons.

### 2.1.2 Next Steps

The development of this level of the framework is largely complete and therefore not the focus of immediate future effort. However, there are two specific possibilities being considered within the context of this project to address the issues mentioned above, should they pose a sufficient hindrance to other aspects of SuperOPF development.

The first is to implement a pure MATLAB® version of MATPOWER's primal-dual interior point solvers, with proper handling of user supplied equality constraints. This would address both the MEX file maintainability issue as well as, hopefully, improve the ability of the algorithm to solve cases containing such constraints.

The second is to evaluate the third party open source optimizer Ipopt [5], recommended by Michael Ferris of University of Wisconsin. This solver may be an improvement in terms of robustness and performance of the algorithm employed, but unfortunately, our initial attempts at evaluation were hindered by the same sort of MEX file hassles encountered with the currently available MATPOWER OPF solvers.

In addition, a separate project will explore the application of advanced optimization algorithms, including Trust-Tech methodologies, to the SuperOPF with the purpose of improving the scalability of the SuperOPF to large problems with a large set of contingencies.

## 2.2 Level 2 – Stochastic Co-optimization Framework

Built on top of MATPOWER's extensible OPF is a stochastic co-optimization framework. The basic concept behind this framework involves the replication of the standard OPF formulation for multiple scenarios. Each scenario is treated as a separate island in a single large network. The standard OPF problem is then formulated for this large unified system, including the standard OPF variables, constraints and costs for each island. The costs are weighted by the scenario probabilities and all of the variables are available to impose additional costs and constraints. For each specific problem formulation, additional variables, contraints and costs are defined which couple the scenarios together into a single stochastic co-optimization problem.

### 2.2.1 Current Status

The current SuperOPF implementation is based on a formulation for a single-period co-optimization of energy, reactive supply and locational reserves over a single base case and a set of credible contingencies. It has a consistent two-stage structure with a stage 1 day-ahead optimization followed by a compatible stage 2 real-time redispatch. The solvers for each stage are currently implemented as a single large OPF problem with the base and contingency scenarios as islands linked by ramp rate limits and redispatch and reserve costs and constraints. MATPOWER's extensible OPF solver is used to solve the unified problem as a single large system. Figure 2 illustrates the structure for an example with a base case and three contingencies.

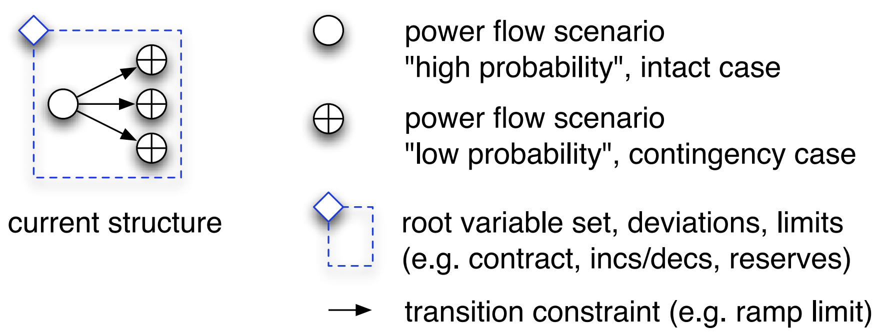


Figure 2: Schematic of Structure of Current Implementation

### 2.2.2 Next Steps

There are a number of directions in which the problem formulation used by the current implementation can be extended and generalized, and this will be the focus of the work for the coming year. The work on the framework structure will focus on the following areas.

- Generalize the linking structure between scenarios in a single time period to allow for multiple base cases.

- Add a time dimension for horizon planning with intertemporal constraints to define capabilities such as load following or storage.
- Add binary decision variables for unit-commitment type formulations.
- Articulate a problem formulation for optimal investment in generation and, if possible, transmission.

The goal is also to define the enhanced framework structure in a more modular fashion, allowing specific problem formulations to be specified at a higher level than that of individual variables, constraints and costs and thereby simplifying both communication and implementation. This will increase in importance as the SuperOPF framework is employed in a broader range of applications, beyond the current context of day-ahead scheduling and real-time redispatch. Specifically, we plan to show how the framework can be applied to the problem of optimal investment in generation.

Decomposition and parallelization strategies and price coordination techniques will also be explored as needed to deal with the increased complexity and size of the new expanded formulations.

A number of relatively minor enhancements to the current software may be possible, such as adding a DC version for use in a planning context. The implementation of the second generation framework, with multiple base cases per period, horizon planning and unit-commitment, on the other hand, is a longer term effort. The focus of the coming year will be on mapping out a modular software architecture that corresponds to the higher level specification of the problem formulation. The purpose of this approach is to both simplify the structure of the code and facilitate the implementation of customized problem formulations for additional applications.

## 2.3 Level 3 – Simulation Infrastructure

Ongoing case studies are an integral part of other projects in this program. Support of those case studies requires the development and maintenance of a simulation infrastructure used to run the case studies and catalog, analyze and visualize the results.

### 2.3.1 Current Status

The simulation infrastructure used for the initial round of cases studies involved simulating a set of “typical hours”, possibly representing a series of peak summer hours for consecutive years of load growth or load slices based on a load duration curve. The latter might be used to explore expected annual costs, revenues, etc.

For each run, representing a single independent time slice (typically an hour), several optimization problems are solved. First the stage 1, or day-ahead, problem is solved to determine energy contracts and reserve allocations. This gives the best day-ahead plan from an expected cost standpoint. Then, to analyze the full range of realizations, the stage 2 re-dispatch problem is solved for the base case and each of the contingencies. So, a full run of a single "hour" in a typical simulation of our 30-bus system with 18 contingencies involves solving 20 optimization problems.

Case studies often involve solving somewhere on the order of 1000's of hours, resulting in tens of thousands of optimization runs. An infrastructure has been developed to automate the set-up and running of the simulations along with the saving of the enormous amounts of data generated.

Due to the nature of the problems being solved and the limitations of our current solvers, a given optimization may not converge successfully within the maximum number of iterations allowed. For this reason, the code is set up to attempt the "best" solver first. If that one fails, it will automatically try the next in line. For stage 1 and the stage 1 base case, we start with PDIPM; for the stage 2 contingency cases, MINOS seems to give a better success rate.

Another issue affecting convergence is the fact that the stage 1 solution is "tight", meaning that the amount of contracted reserve is just enough to survive the worst contingencies. This can cause problems since the numerical solutions are not exact. We address this issue by relaxing the problem "tightness" by contracting for $\epsilon$ of "extra" reserves. If the stage 2 problem fails, the $\epsilon$ is increased (up to some limit) and automatically retried.

The simulation is also programmed to automatically skip over runs that have already been successfully completed, to allow for restarting a run where it left off in the case of an interruption.

As an aid in analyzing the results, a number of automated routines have been developed to generate standard visualizations and statistics.

### 2.3.2 Next Steps

As other case studies are designed, the simulation infrastructure will be adapted and extended as needed to accomodate the new requirements. One planned modification is move from simulating independent "typical" hours to a set of sequential hours in a typical day or week. This will likely involve saving data from the output of one period and using it to initialize parameters in the formulation of the next or subsequent periods.

For more complex problems, such as horizon planning, involving a simultaneous optimization of multiple time periods, a decomposition approach may be advantageous. This would require an additional layer to take advantage the possibility of running the simulations in parallel on a computing cluster.

# References

[1] MATPOWER: http://www.pserc.cornell.edu/matpower/.

[2] MINOPF: http://www.pserc.cornell.edu/minopf/.

[3] TSPOPF: http://www.pserc.cornell.edu/tspopf/.

[4] H. Wang, C. E. Murillo-Sánchez, R. D. Zimmerman, and R. J. Thomas, "On computational issues of market-based optimal power flow," *Power Systems, IEEE Transactions on*, vol. 22, no. 3, pp. 1185–1193, August 2007.

[5] Ipopt: https://projects.coin-or.org/Ipopt/.